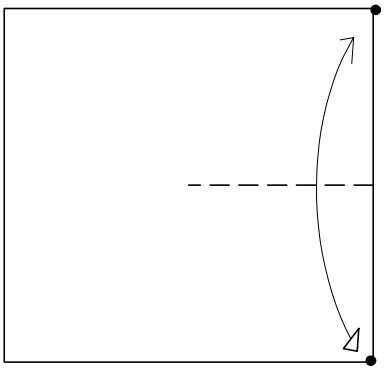
The Einstein Hat

Designed from 3/27/23-3/31/23; Diagrammed from 4/1/2023 to 4/3/2023.
This 13-sided polygon is the first known example of an "einstein," a single shape that can be tiled to cover a plane with a pattern that never repeats. The shape was discovered by David Smith.
After several failed attempts on the modular approach using either a 4-unit diamond shape or an 8-unit kite shape, I put them aside and started to work on a single-piece version. Finally, I came up with this one-piece solution.

1. Start from a square with both sides of the same color. White used here for clarity.



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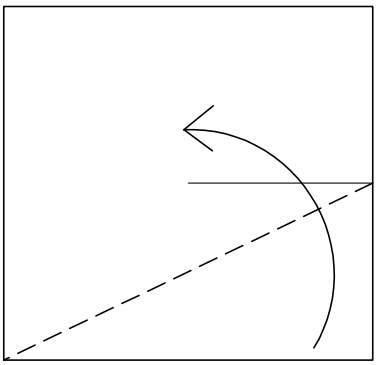
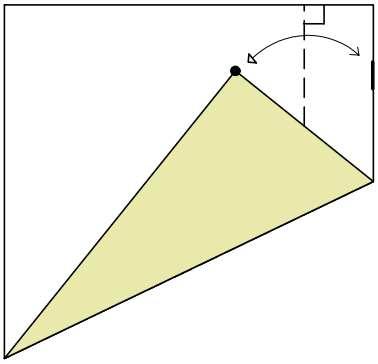


3. Mark a crease by folding edge to point. Unfold the diagonal flap afterward.



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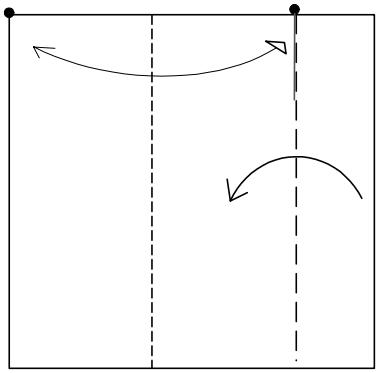

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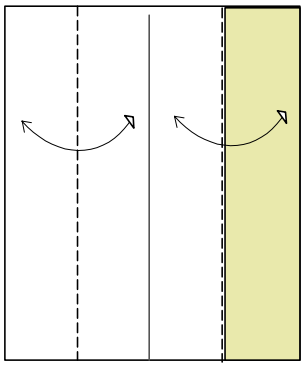

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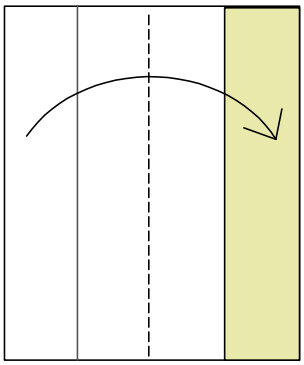
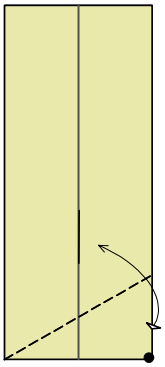


7. Fold point to center line.



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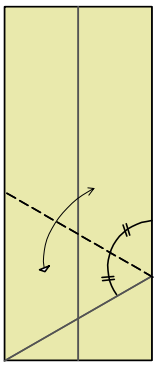

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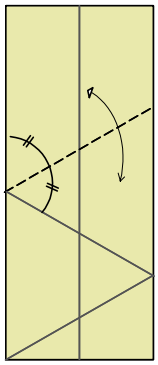

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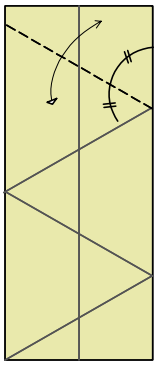
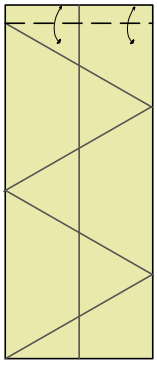


11. Unfold the paper back to the original square after making the upper horizontal crease.



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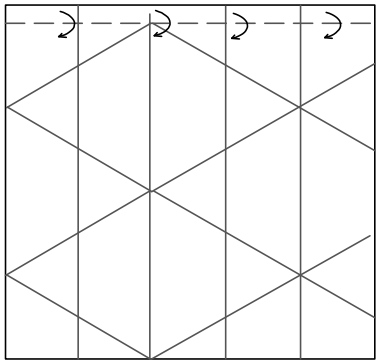

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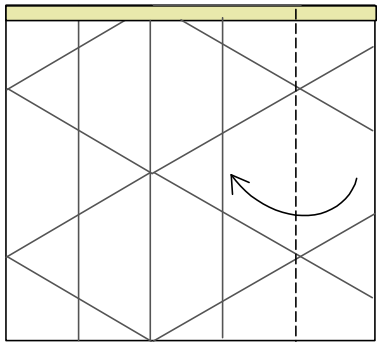

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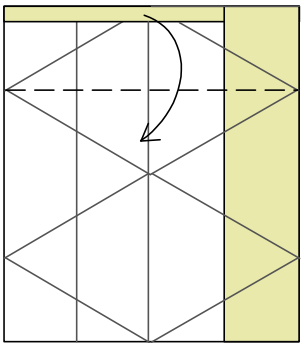

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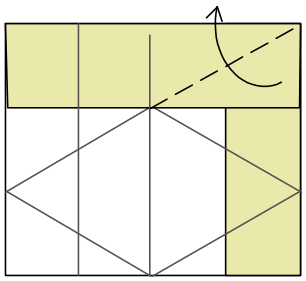
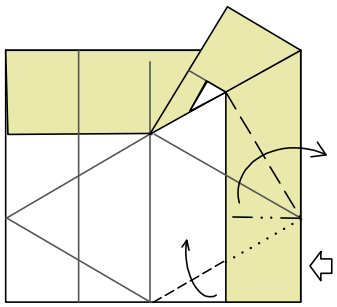


16. Squash the lower right corner.



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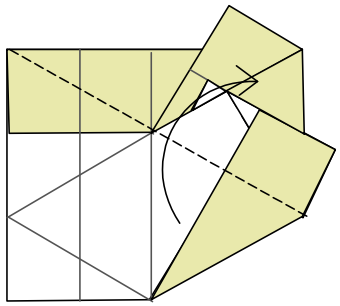
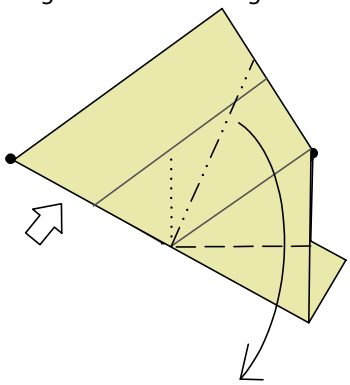
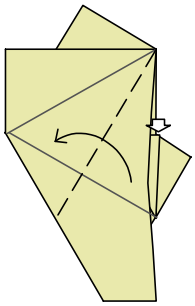


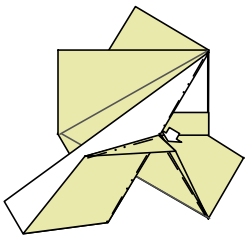
18. Fold the upper flap down while squashing the lower left edge. Two dots will align.



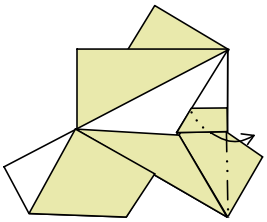
19. Fold the lower flap to the left while squashing the hidden edge beneath.



20. In-progress view; Squash to flatten the model.



21. Pull out the single trapped layer near the lower right corner.



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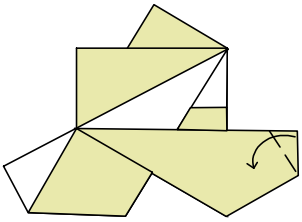

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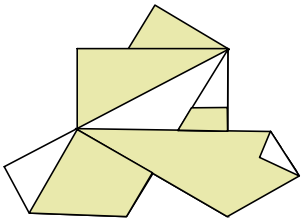
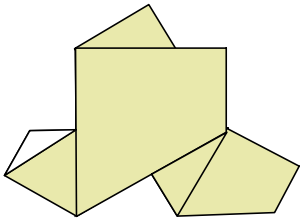
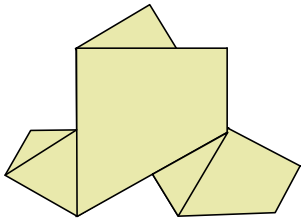


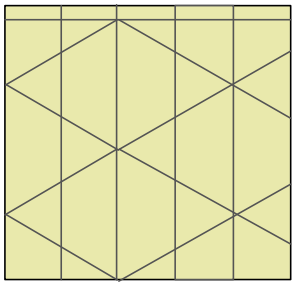
24. The hat completed



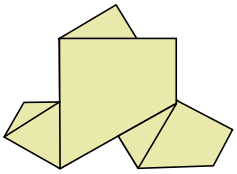
The monocolor version



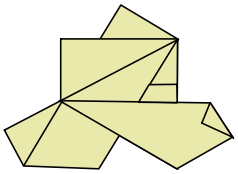
Starting crease pattern



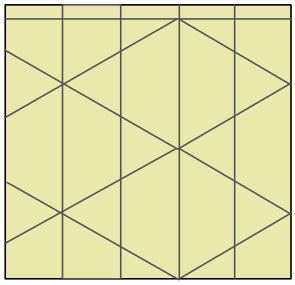
Cleaner side

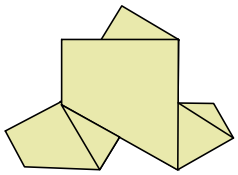


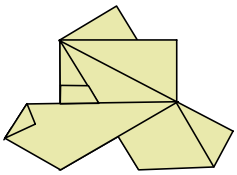
The other side



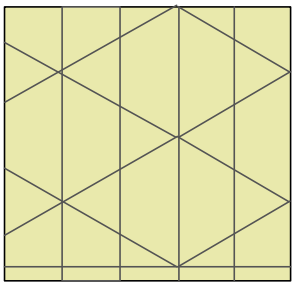
Mirrored crease pattern

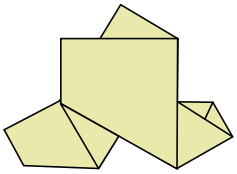


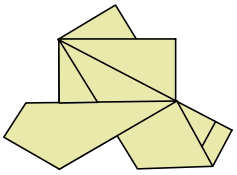




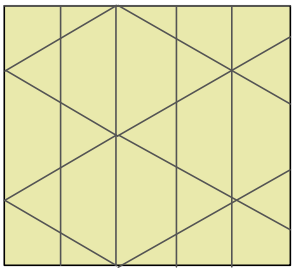
Folded strip at the bottom

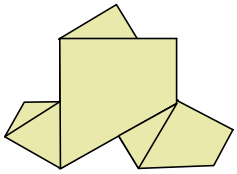


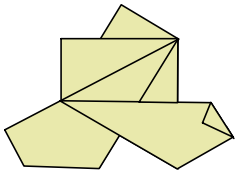




Redundant strip cut off



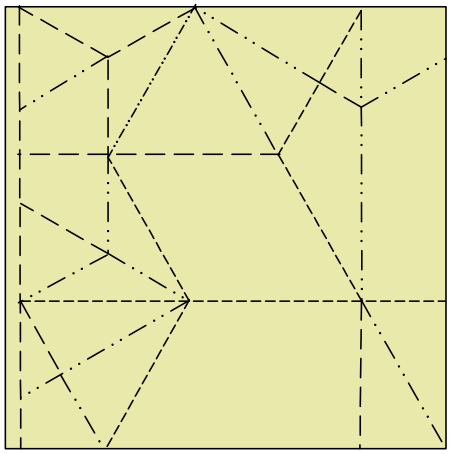




The Einstein Turtle

Smith later discovered another einstein shape that looked like a turtle!
It happens to be another 13-sided polygon but consisting of 10 kites connected at their edges. After folding many pieces of hats, I figured the new turtle shape would be a good challenge. Here is my crease pattern for the turtle.

Crease pattern



The 13-sided turtle from 10 connected kites